

# On the $L_x - \sigma_v$ relation of groups of galaxies

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**Abstract.** We analyse the  $L_x - \sigma_v$  relation for the new Mulchaey et al. group Atlas. We find that once we take into account the possible statistical bias introduced by the cutoff in luminosity, we recover a relation which is consistent with that of clusters, ie.,  $L_x \propto \sigma_v^{4.1}$ . The larger scatter of this relation for groups of galaxies could be attributed to an orientation effect, due to which the radial velocity dispersion of groups oriented close to orthogonal to the line of sight, would be underestimated. This effect could also contribute in the direction of flattening the slope of the group  $L_x - \sigma_v$  relation.

**Key words:** galaxies: clusters : general – X-rays: galaxies

## 1. Introduction

Most galaxies in the universe occur in small groups (cf. Geller & Huchra 1983; Tully 1987; Nolthenius & White 1987), which in many respects could be considered as poor clusters of galaxies. The detection of X-ray emission from some groups has increased considerably the interest to study them since they proved to be real entities and not projection effects (for an extensive review see Mulchaey 2000). Solinger & Tucker (1972) showed that if the source of the X-ray emission is hot gas bound in clusters, then the X-ray luminosity,  $L_x$ , should be correlated with the optical radial velocity dispersion,  $\sigma_v$ . Simple theoretical arguments show that in a virialized, isothermal aggregation of gas, which emits thermal bremsstrahlung emission ( $L_x \propto \int \rho_{gas}^2 T(r)^{1/2} dV$ ), we can obtain that  $L_x \propto M^{4/3}$  (where  $M$  is the total of the system) and using virial arguments ( $M \propto \sigma_v^3$ ) we then have that  $L_x$  should be roughly proportional to the fourth power of  $\sigma_v$ :  $\log L_x \propto \log \sigma_v^4$  (cf. Navarro, Frenk & White 1995). Quintana & Melnick (1982) first showed that the X-ray luminosity of clusters of galaxies obey the expected correlation.

Numerical simulations have shown that the relationship between  $L_x$  and  $\sigma_v$  for groups should be similar to that of clusters (cf. Navarro, Frenk, & White 1997), or even steeper if one takes into account radiative cooling

which significantly reduces the amount of the hot gas fraction at low- $\sigma_v$ 's (Davé, Katz & Weinberg 2002). Such a steep relation for systems with  $L_x < 10^{43}$  ergs s<sup>-1</sup>, has been advocated by Mahdavi & Geller (2001), although they notice an erratic behavior of the poor groups of galaxies.

In some observational studies a consistency has been found between the  $L_x - \sigma_v$  relation of groups and clusters with a slope  $\sim 4$  (cf. Ponman et al 1996; Mulchaey & Zabludoff 1998; Helsdon & Ponman 2000), while in others a shallower slope has been found, i.e., groups appear to have a relatively enhanced X-ray emission to what predicted from the  $L_x - \sigma_v$  relation deduced from clusters of galaxies (cf. Dell'Antonio et al. 1994; Mahdavi et al. 1997, 2000; Xue & Wu 2000). Attempts to explain such deviations of the group from the cluster  $L_x - \sigma_v$  behaviour have invoked a possible contribution of individual galaxy halos to the group X-ray luminosity (Mahdavi et al. 2000), poorly determined  $\sigma_v$ 's and/or  $L_x$ 's (Zimer, Mulchaey & Zabludoff 2001), a large scatter due to a non-equilibrium galaxy velocity distribution (Mahdavi & Geller 2001).

An alternative explanation, based on a possible orientation effect, was proposed by Tovmassian, Tiersch, & Yam (2002) [see also Tovmassian, Martinez, & Tiersch 1999; Tovmassian 2002]. They considered the relatively small RASSCALs and HCG X-ray group samples (Mahdavi et al. 2000; Ponman et al. 1996, respectively) and showed that the flattening of groups with relatively small velocity dispersions is, on average, larger than those of groups with higher velocity dispersions. They suggested that the shallow shape of  $\log L_x - \log \sigma_v$  of groups of galaxies could be partly the result of an underestimation of the velocity dispersion of elongated groups when seen roughly orthogonally to the line of sight, while when seen edge-on, they will have higher and probably nearer to their true  $\sigma_v$  values. This correlation could be explained if member galaxies in groups move preferentially along the group elongation, since groups have been found to have a prolate-like shape (Hickson et al. 1984; Malykh & Orlov 1986; Oleak et al. 1998) as in the case of clusters (cf. Plionis, Barrow & Frenk 1991).

Recently Mulchaey et al. (2003) published an X-ray Atlas of groups of galaxies, which is the largest sample of groups studied to date having X-ray ROSAT PSPC pointed observations. In this paper we address two questions: is there sufficient evidence to support claims that groups have enhanced, with respect to clusters, X-ray emission (ie., that the power law index of the  $\log L_x - \log \sigma$  relation has  $\alpha < 4$ ) and what is the origin of the larger scatter of the  $L_x - \sigma$  relation for groups.

## 2. Data and results

In order to address the issue of the possibly enhanced group X-ray emission we have selected from the Atlas of Mulchaey et al (2003) those with detected X-ray emission that contain less than 20 members (since a larger membership should rather define poor clusters). Note that the  $L_x$  values were determined after the removal of point sources. We exclude three groups: NGC 2484 and NGC 6251 because they consist of only two members and the NGC 6329 group due to its very large projected length  $a$  ( $\approx 11\text{Mpc}$ )<sup>1</sup>, which is suspected that it could be a superposition of two or even more groups and hence, its  $\sigma_v$  value may be unreliable. We are left with 43 groups in total out of which we have three triplets. Although, Focardi & Kelm (2002) argue that triplets are a distinct class of low- $\sigma_v$  objects, in our case they span the whole  $\sigma_v$  range.

In Fig. 1 we plot  $\log L_x$  versus  $\log \sigma_v$  for the considered groups (open symbols) and for groups with members  $N > 20$  (squares), which could be considered as poor clusters. The latter are closely located along the line  $L_x \propto \sigma_v^4$  (solid line).

We perform a direct least-square regression to the  $N < 20$  groups of the type

$$\log L_x = \alpha \log \sigma + C_1 \quad (1)$$

and find a strong correlation with a correlation coefficient  $R = 0.65$  and a probability of random occurrence  $3 \times 10^{-6}$ . The fitted parameters are:

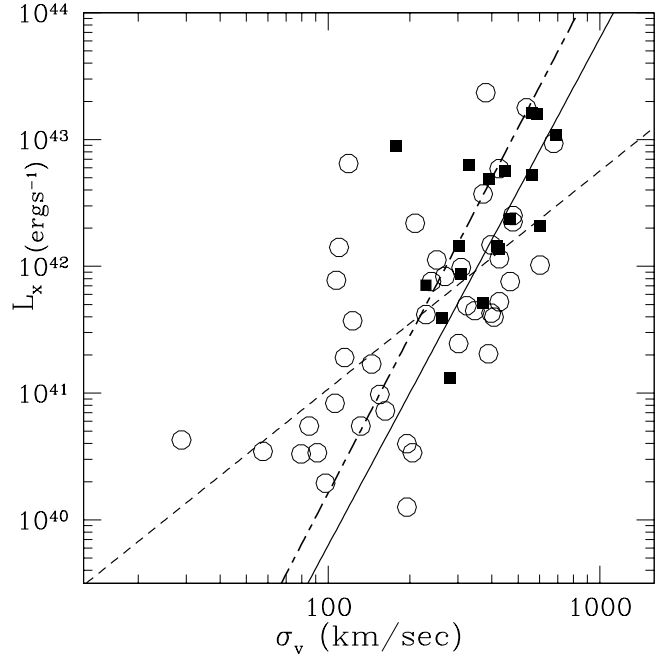
$$\alpha = 1.72 \pm 0.31 \quad C_1 = 37.6 \pm 0.75 .$$

The determined slope is much shallower than the expected value for clusters  $\alpha \simeq 4$ , while the value found from the  $N > 20$  groups is  $\alpha \simeq 3.2$ . As discussed in the introduction, the shallower slope of the group  $L_x - \sigma_v$  relation already been noticed in other studies as well and indeed it has stimulated speculations on the reason for the apparently enhanced X-ray emission of groups. Hence, we could have concluded from our results that this sample supports the claims for a relatively enhanced X-ray emission for groups.

### 2.1. Statistical bias ?

However, what we are witnessing is the result of a statistical bias, resembling the Malmquist bias, which appears

<sup>1</sup> Lengths are determined assuming  $H_0 = 72 \text{ km s}^{-1}\text{Mpc}^{-1}$ .



**Fig. 1.**  $L_x - \sigma_v$  relation for groups: those with  $N \leq 20$  are represented as circles, while those with  $N > 20$ , supposed to be poor clusters, are marked by filled squares. The solid line is the  $\log L_x \propto \log \sigma_v^4$  relation of clusters of galaxies. The dashed line is the best direct regression fit for the groups ( $\log L_x \propto \log \sigma_v^{1.7}$ ) and the dot-dashed line is the best inverse regression fit for the groups ( $\log \sigma_v \propto (1/4.1) \log L_x$ ).

because of the low- $L_x$  limit (either due to the lower mass limit necessary for the ICM to light up, or due to the X-ray flux limit in the construction of the sample). This bias is of the same nature that enters in scaling relation (eg. Tully-Fisher, Faber-Jackson, etc) where the magnitude or flux limit imposes a bias such that the slope of the derived relation is shallower than the nominal one. The larger the scatter of the relation, the larger the bias imposed. This bias may or may not appear in the corresponding cluster relation depending on the amplitude of the scatter around the nominal relation. To understand this bias let us imagine that the cluster relation  $L_x \propto \sigma^4$  is obeyed by groups as well and let us consider a subset of our sample of groups that have  $\sigma_v \simeq 600 \text{ km s}^{-1}$  so that they are typically quite brighter than the lower  $L_x$  limit. It is clear that their  $\log L_x$  value will be distributed around the mean  $\langle \log L_x \rangle$  value with some dispersion  $\sigma$ . However, if we take a subsample with  $\sigma_v \simeq 100 \text{ km s}^{-1}$  so that they are on average quite faint with  $\langle \log L_x \rangle \simeq \log(L_x)_{limit}$ , then the only groups that will appear in the sample are those with  $\log L_x > \langle \log L_x \rangle$  and none with  $\log L_x < \langle \log L_x \rangle$ . This will induce the above mentioned bias.

To see this more clearly, we have performed Monte-Carlo simulations in which we assume a relation  $L_x \propto \sigma_v^4$ ,

and a Gaussian scatter of  $\delta \log L_x = 0.8$ . We then perform a direct (ie., eq.1) and an inverse least-square regression fit to the resulting data (ie., we fit  $\log \sigma_v = (1/\alpha) \log L_x + C_2$ ). In Fig. 2a we show such a simulation with 5000 “groups”, where the solid line is the input  $L_x \propto \sigma^4$  relation and the dashed line is the recovered from the direct regression. The inverse regression recovers exactly the input slope.

Furthermore, to study more accurately the effect of this statistical bias on our sample we have performed a series of simulations in which we use the observed  $L_x$  values of the groups and we derive the values  $\sigma_v$ , assuming a Gaussian scatter of the  $\log L_x - \log \sigma_v$  relation of varying magnitude. We have performed 1000 such simulations for each different value of the Gaussian scatter. In Fig. 2b we plot the derived values of the slope  $\alpha$  for both regression methods as a function input scatter. The vertical line indicates the scatter of the relation derived from the  $N \leq 20$  sample of groups. The direct regression method (star-like symbols) underestimates severely the input slope, with the underestimation increasing with increasing scatter, while the inverse regression method (open symbols) recovers it accurately. Note also that we plot (squares) the results of the inverse regression in case that there is a sharp cutoff at  $\sigma_v = 1000 \text{ km s}^{-1}$ , in which case a similar, although of smaller magnitude, bias is introduced in the opposite direction.

It is evident that:

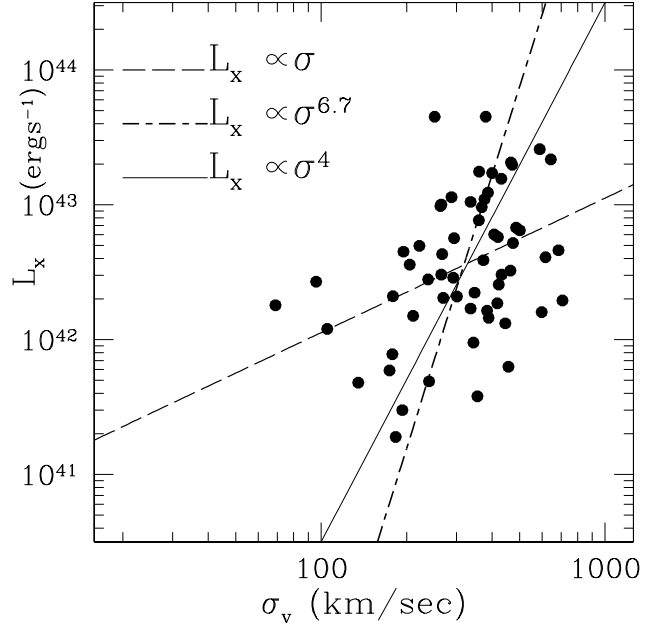
- The inverse regression recovers correctly the input slope of the  $\log L_x - \log \sigma_v$  relation for all values of the scatter.
- for the observed amount of scatter ( $\delta C_1 \simeq 0.8$ ) the expected slope of the relation once we use the direct regression is around  $\sim 2$ , as indeed observed.

As a further test of this bias, we have re-analysed the group data of Xue & Wu (2000). Their sample consists of 60 groups that have velocity dispersion and X-ray luminosity data. Performing the direct regression fit we recover their results (listed in their table 3 - OLS method), ie.  $\alpha \simeq 1$  and  $C_1 \simeq 40$ . However, if we perform an inverse regression fit to their data we recover a very different slope, ie.,  $\alpha \simeq 6.7$  and  $C_1 \simeq 25.7$ . In Fig.3 we plot the  $L_x - \sigma$  correlation for this sample and the fitted lines for both regression methods. Using our Monte-Carlo procedure we have seen that such a dichotomy between the direct and inverse regression methods can be approximately accommodated if there is a scatter of  $\sim 1$  in  $\log L_x$  and a cutoff at  $\sigma \simeq 600 \text{ km s}^{-1}$ .

Guided by our Monte-Carlo analysis we performed an inverse regression to our sample of  $N \leq 20$  groups, and we have indeed recovered a slope

$$\alpha \simeq 4.1 \pm 0.6 .$$

We therefore conclude that this sample of X-ray groups is absolutely consistent with the relation found from clusters of galaxies.



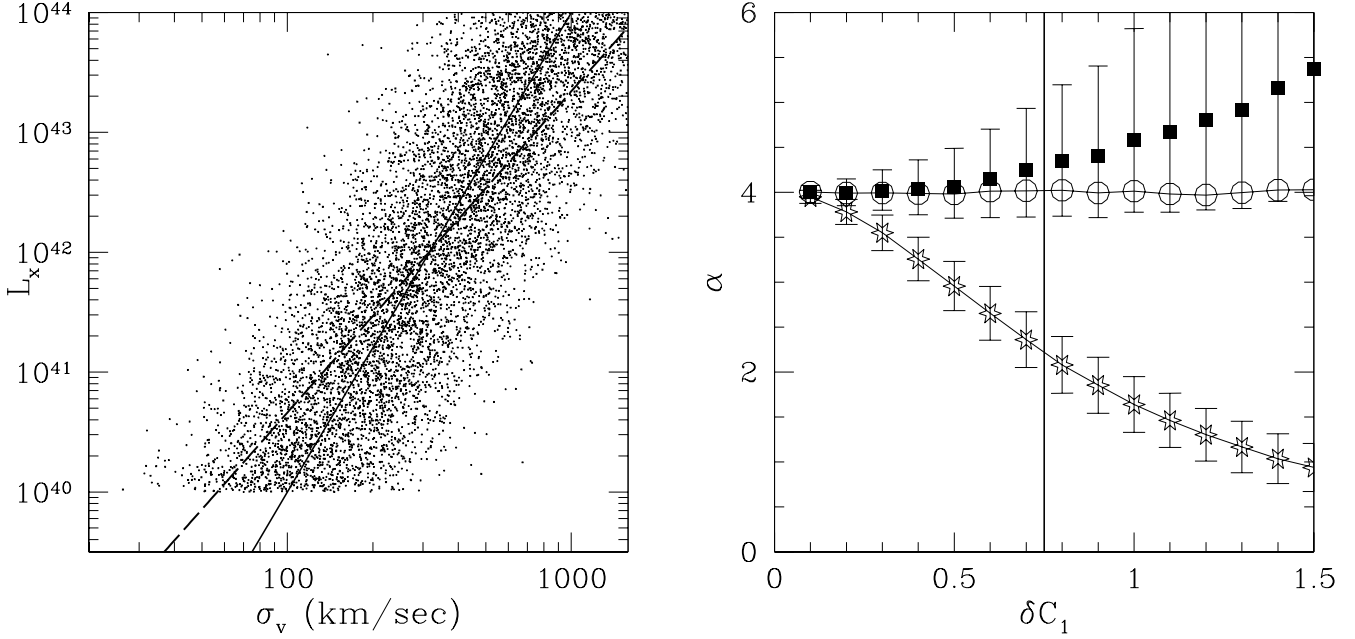
**Fig. 3.** Results of the Xue & Wu (2000) sample of groups.

## 2.2. The scatter in the $L_x - \sigma$ relation

Now we move to address the issue of the larger scatter of the  $L_x - \sigma_v$  relation of groups than of clusters. This is an important issue because, as we have seen previously, the larger scatter will induce an artificial flattening of the  $L_x - \sigma_v$  relation.

We will show below that the larger scatter is not only statistical, due to possible measurement uncertainties especially in the low- $L_x$  and  $\sigma_v$  regime, but has a probably large intrinsic component which is due to an orientation effect. To guide the reader through our arguments we need first to define the axis ratio  $b/a$  of the groups studied<sup>2</sup>. The ratio  $b/a$  was determined by using the positions of member galaxies which are mentioned in the corresponding references of Table 1 in Mulchaey et al. (2003). For those groups that Mulchaey et al. (2003) used the NASA Extragalactic Database (NED), we selected the members of the corresponding groups by their redshifts taking into account the membership number, mentioned in Mulchaey et al. (2003). Note that that member galaxies are not drawn from a well-defined and uniform magnitude limited sample. This can introduce differences in the depth coverage within groups, but such an effect would probably introduce a random error in the determination of the

<sup>2</sup>  $a$  is the angular distance between the most widely separated galaxies in the group, and  $b$  is the angular distances  $b$  of the third galaxy of the group consisting of three galaxies from the line  $a$  joining the most separated two galaxies, or is the sum of the angular distances  $b_1$  and  $b_2$  of the most distant galaxies on either side of the line  $a$  joining the most separated galaxies (Rood 1979).

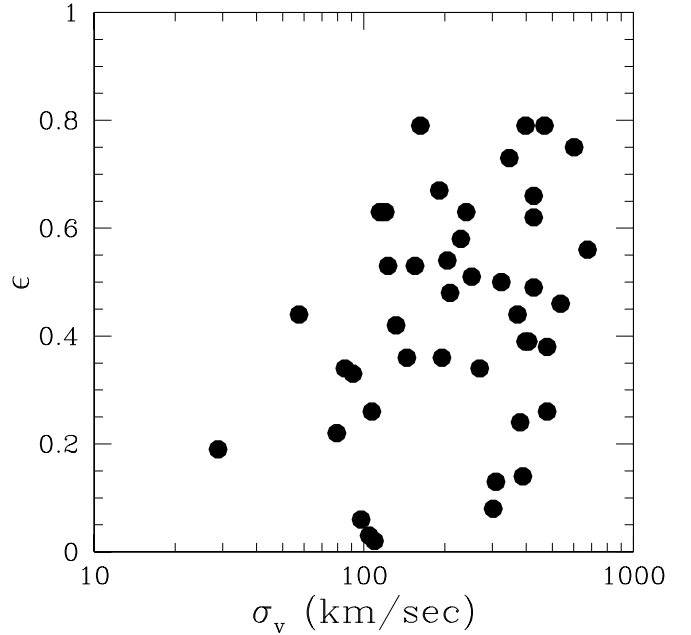


**Fig. 2.** (a) Manifestation of the statistical bias for 5000 “groups” that have been drawn from a  $L_x \propto \sigma_v^4$  relation (solid line), which have a scatter of 0.8 in  $\log L_x$ . The recovered direct regression line is the dashed one. (b) Recovered values of the  $L_x - \sigma_v$  slope ( $\alpha$ ) for different amounts of scatter of the relation for the direct regression ( $\log L_x \propto \alpha \log \sigma_v$ ; star symbols), for the inverse regression ( $\log \sigma_v \propto (1/\alpha) \log L_x$ ; open symbols) and for the inverse regression once we have imposed an upper limit in  $\sigma_v = 1000 \text{ km s}^{-1}$ .

group elongation. In Table 1 we present  $b/a$  values for those groups with number of members  $N < 20$ . This is the sample we will investigate in detail.

We have noticed that there is a fairly significant correlation ( $R = 0.37$  and  $\mathcal{P} = 0.025$ ) between flattening and velocity dispersion, with flatter systems having lower velocity dispersions (see Fig. 4). Taking the 10 groups with the lowest and the highest values of  $\sigma_v$  ( $\log \sigma_v \leq 2.07$  and  $\geq 2.6$ , respectively) we find that the median and 68% quantiles of their  $b/a$  ratio is  $0.22^{+0.04}_{-0.16}$  and  $0.49^{+0.07}_{-0.10}$ , respectively.

This difference in the group flattening is also accompanied by a significant difference in the number of group members, with mean values of  $\sim 5.2$  and  $9.2$  respectively. This could be expected either (a) due to the number of galaxies - mass correlation (higher number of galaxies implies higher mass which then implies a higher velocity dispersion) or (b) due to the fact that sparser groups are more likely, merely by chance, to show higher elongations than more dense systems. The latter possibility has been ruled out after performing a large set of Monte-Carlo simulations in which we have generated the same number of groups and group members, as in the two observed subsamples, by randomly placing group members in a sphere and then projecting them to a plane. The resulting median axial ratio for the low and high- $\sigma$  subsamples was found to be:  $\sim 0.62 \pm 0.18$  and  $\sim 0.7 \pm 0.15$ ,



**Fig. 4.** Group axial ratio versus group  $\log \sigma_v$ .

respectively, significantly larger than the observed values. However, neither the former possibility can explain the significant difference in the flattening of the two extreme

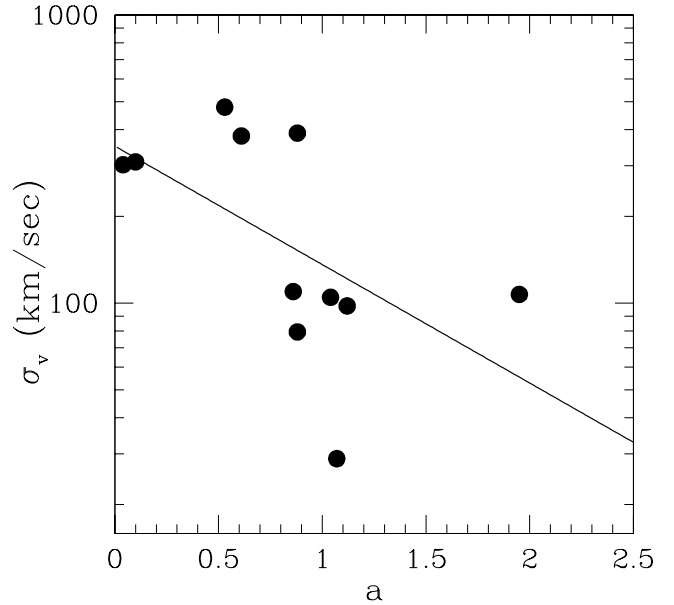
**Table 1.** The  $b/a$  ratios and the number of galaxy members for groups with  $2 < N < 20$ .

Group	N	$b/a$	Group	N	$b/a$	Group	N	$b/a$	Group	N	$b/a$
NGC 315	4	0.53	NGC 1407	8	0.36	NGC 4065	7	0.38	HCG 68	5	0.53
NGC 326	9	0.56	NGC 1587	4	0.03	NGC 4104	8	0.66	ARP 330	8	0.44
NGC 524	8	0.54	NGC 2300	13	0.58	NGC 4125	3	0.44	NGC 6338	11	0.46
HCG 12	5	0.63	HCG 37	5	0.39	NGC 4261	8	0.26	HCG 90	16	0.67
NGC 720	4	0.79	HCG 48	3	0.08	SHK 202	5	0.75	UGC 12064	9	0.79
HCG 15	6	0.49	CGCG 154-041	4	0.13	NGC 4291	11	0.42	HCG 92	4	0.14
HCG 16	9	0.34	NGC 3607	7	0.33	NGC 4636	12	0.79	IC 1459	5	0.22
UGC 1651	3	0.02	NGC 3647	6	0.24	NGC 5044	9	0.63	NGC 7619	7	0.51
NGC 1044	13	0.50	NGC 3665	4	0.19	NGC 5171	15	0.26	HCG 97	14	0.62
IC 1880	7	0.48	HCG 57	7	0.34	HCG 67	14	0.73	NGC 7777	4	0.63
UGC 2775	5	0.39	NGC 3923	5	0.06	NGC 5322	8	0.36			

subsamples of groups. Why should poorer groups be flatter than richer ones? A possible explanation could be the different level of group virialization. If groups accrete material anisotropically along one dimensional structures, like filaments, then one may expect that flatter systems are dynamically younger (thus they have lower values of  $\sigma_v$  and  $L_x$ ) while when virialization takes place, after the groups have accreted enough material, it will drive groups to more spherical configurations, with higher velocity dispersions.

This straight-forward explanation cannot account however for all the observables. For example, selecting only the groups with  $N \leq 5$  members (in total 10 groups), which as expected from the above discussion should be very flat (indeed their median  $b/a$  is  $0.14^{+0.05}_{-0.06}$ ), we find that although in this subsample there is no correlation of  $\sigma_v$  with the number of group members, there is a significant correlation between their flattening and  $\sigma_v$  (correlation coefficient  $R=0.8$  and  $\mathcal{P}_{\text{random}} = 0.006$ ), with low- $\sigma_v$  groups being flatter. In the virialization paradigm discussed above such a correlation cannot be explained. In order to explain such correlations we suggest that an *orientation effect* is at work, i.e., if galaxy members move along their group major axis (for example, infalling in their common center of mass or rotating in elongated orbits around their gravitational center; cf. Tovmassian 2001, 2002), then flat groups oriented close to orthogonally to the line of sight (small  $b/a$ ) will have small values of  $\sigma_v$ , while the opposite is true for flat groups seen edgewise.

According to this view the length of the apparent major axis,  $a$ , of the flattest groups (small  $b/a$  ratio) should also depend on orientation. Groups oriented close to the line of sight will have on average small  $a$  and high  $\sigma_v$ , and groups oriented close to the orthogonal to the line of sight will have large  $a$  and small  $\sigma_v$ . Tovmassian (2002) showed that such anti-correlation between the  $a$  and  $\sigma_v$  is observed in the case of HCGs groups. In order to investigate this possibility in our sample, we select the 11 groups with  $b/a < 0.35$ . In Fig. 5 we plot their apparent value of  $a$  versus  $\sigma_v$ . The expected trend is indeed apparent,

**Fig. 5.** The flat ( $b/a < 0.35$ ) group  $a - \log \sigma_v$  correlation.

with  $a$  increasing as  $\sigma_v$  decreases, and has a probability of occurring randomly of  $\mathcal{P} = 0.07$ . Performing a bootstrap resampling technique in order to estimate the uncertainty of the above significance level, we find  $\delta\mathcal{P} = 0.02$ .

The existence of the orientation effect does not imply that the virialization arguments, discussed previously, are incorrect. Most probably both are at work since the orientation effect is not apparent in groups with large number of members, exactly because these groups are richer and most probably in a more advanced dynamical state.

### 3. Conclusions

We have investigated the X-ray luminosity - velocity dispersion relation in the new group Atlas of Mulchaey et al. (2003). A direct regression shows that there is a strong correlation with a slope ( $\alpha \simeq 1.7$ ) significantly shallower than

that found in clusters of galaxies. However, we attribute this to a statistical bias, resembling the Malmquist bias, that enters in the direct regression approach and which is due to a limit in  $L_x$ . We have quantified this using Monte-Carlo simulations and once we use the more accurate inverse regression method we obtain a slope of  $\sim 4.1$ , absolutely consistent with the cluster relation. We have also investigated the apparently larger scatter of the  $L_x - \sigma$  relation of groups with respect to clusters. We find that at least part of this scatter is intrinsic in nature and due to an orientation effect by which flat groups seen edge-on have their  $\sigma_v$  values underestimated. This can be understood by noting that groups of galaxies have a roughly prolate spheroidal shape and thus if member galaxies move along their major axis, being accreted for example to their common centre of mass, then the correlation between  $\sigma_v$  and their major axis, could be due to the different group orientations with respect to the line of sight. Flat groups that are oriented roughly orthogonally to the line of sight will have low values of  $\sigma_v$ , while when oriented close to the line of sight or at intermediate angles, they will have higher values of  $\sigma_v$ .

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